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Starlikeness of integral transforms and duality ☆

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ABSTRACT

For λ satisfying a certain admissibility criteria, sufficient conditions are obtained that ensure the integral transform

$$V_\lambda(f)(z) := \int_0^1 \lambda(t)\frac{f(tz)}{t}\,dt$$

maps normalized analytic functions f satisfying

$$\operatorname{Re} e^{i\phi}\left((1-\alpha+2\gamma)\frac{f(z)}{z} + (\alpha-2\gamma)f'(z) + \gamma z f''(z) - \beta\right) > 0$$

into the class of starlike functions. Several interesting examples of λ are considered. Connections with various earlier works are made, and the results obtained not only reduce to those earlier works, but indeed improved certain known results. As a consequence, the smallest value $\beta < 1$ is obtained that ensures a function f satisfying $\operatorname{Re}(f'(z) + \alpha z f''(z) + \gamma z^2 f'''(z)) > \beta$ is starlike.