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Fixed point solutions of variational inequalities for asymptotically nonexpansive mappings in Banach spaces

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Abstract

Let E be a real Banach space with a uniformly Gâteaux differentiable norm and which possesses uniform normal structure, K a nonempty bounded closed convex subset of E, $T : K \longrightarrow K$ an asymptotically nonexpansive mapping with sequence $\{k_n\}_n \subset [1, \infty)$. Let $\{t_n\} \subset (0, 1)$ be such that $t_n \to 1$ as $n \to \infty$ and f be a contraction on K. Under suitable conditions on the sequence $\{t_n\}$, we show the existence of a sequence $\{x_n\}_n$ satisfying the relation $x_n = (1 - \frac{t_n}{k_n}) f(x_n) + \frac{t_n}{k_n} T^n x_n$, and prove that $\{x_n\}_n$ converges strongly to the fixed point of T, which solves some variational inequality, provided $\|x_n - T x_n\| \to 0$ as $n \to \infty$. As an application, we prove that the iterative process defined by $z_0 \in K,\ z_{n+1} := (1 - \frac{t_n}{k_n}) f(z_n) + \frac{t_n}{k_n} T^n z_n,\ n \in \mathbb{N}$, converges strongly to the same fixed point of T.

MSC: 47H06; 47H09; 47J05; 47J25

Keywords: Viscosity approximation; Asymptotically nonexpansive mapping; Variational inequality; Normal structure

1. Introduction

Let E be a real Banach space with dual E^* and K a nonempty closed convex subset of E. Let $J : E \longrightarrow 2^{E^*}$ denote the *normalized duality mapping* defined by $J(x) := \{f \in$

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