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A study of an impulsive four-point nonlocal boundary value problem of nonlinear fractional differential equations

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ABSTRACT

This paper studies the existence and uniqueness of solutions for a four-point nonlocal boundary value problem of nonlinear impulsive differential equations of fractional order $q \in (1, 2]$. Our results are based on some standard fixed point theorems. Some illustrative examples are also discussed.

1. Introduction

Boundary value problems for nonlinear differential equations arise in a variety of areas of applied mathematics, physics and variational problems of control theory. A point of central importance in the study of nonlinear boundary value problems is to understand how the properties of nonlinearity in a problem influence the nature of the solutions to the boundary value problems. Multi-point nonlinear boundary value problems, which refer to a different family of boundary conditions in the study of disconjugacy theory [1] and take into account the boundary data at intermediate points of the interval under consideration, have been addressed by many authors, for example, see [2–9] and the references therein. The multi-point boundary conditions are important in various physical problems of applied science when the controllers at the end points of the interval (under consideration) dissipate or add energy according to the sensors located at intermediate points.

In recent years, differential equations of fractional order have been addressed by several researchers with the sphere of study ranging from the theoretical aspects of existence and uniqueness of solutions to the analytic and numerical methods for finding solutions. Fractional differential equations appear naturally in various fields of science and engineering such as physics, polymer rheology, regular variation in thermodynamics, biophysics, blood flow phenomena, aerodynamics, electrodynamics of complex medium, viscoelasticity, electrical circuits, electron-analytical chemistry, biology, control theory, fitting of experimental data, etc. [10–12]. The recent theoretical developments and applications of fractional differential equations can be found in the texts [13–15]. For some recent results on fractional neutral differential equations, see [16,17]. Unlike the boundary value problems of differential equations of integer order, the theory of fractional boundary value problems is not so rich and many aspects of these problems need to be developed. Some recent work on fractional boundary value problems can be found in [18–31] and the references therein.

Impulsive differential equations, which provide a natural description of observed evolution processes, are regarded as important mathematical tools for the better understanding of several real world problems in applied sciences. In fact, the theory of impulsive differential equations is much richer than the corresponding theory of ordinary differential equations without impulse effects since a simple impulsive differential equation may exhibit several new phenomena such as rhythmical beating, merging of solutions and noncontinuability of solutions. For the general theory and applications of impulsive differential equations, we refer the reader to the references [32–36]. On the other hand, the impulsive boundary value problems for nonlinear fractional differential equations have not been addressed so extensively and many aspects of

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