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Research Article

A Jacobi Dual-Petrov-Galerkin Method for Solving Some Odd-Order Ordinary Differential Equations

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A Jacobi dual-Petrov-Galerkin (JDPG) method is introduced and used for solving fully integrated reformulations of third- and fifth-order ordinary differential equations (ODEs) with constant coefficients. The reformulated equation for the Jth order ODE involves n-fold indefinite integrals for $n = 1, \ldots, J$. Extension of the JDPG for ODEs with polynomial coefficients is treated using the Jacobi-Gauss-Lobatto quadrature. Numerical results with comparisons are given to confirm the reliability of the proposed method for some constant and polynomial coefficients ODEs.

1. Introduction

A well-known advantage of spectral methods is high accuracy with relatively fewer unknowns when compared with low-order finite-difference methods [1, 2]. On the other hand, spectral methods typically give rise to full matrices, partially negating the gain in efficiency due to the fewer degrees of freedom. In general, the use of the Jacobi polynomials ($P_n^{(\alpha,\beta)}$ with $\alpha, \beta \in (-1, \infty)$ and n is the polynomial degree) has the advantage of obtaining solutions of ordinary differential equations (ODEs) in terms of the Jacobi indices (see for instance, [3–5]). Several such pairs (α, β) have been used for approximate solutions of ODEs (see [6–10]). We avoid developing approximation results for each particular pair of indices and instead carry out a study with general indices. With this motivation, we introduce in this paper a family of the Jacobi polynomials with general indices.

Third-order differential equations have applications in many engineering models, see for instance [11–14]). Fifth-order differential equations generally arise in the mathematical